Dual-PQC GAN Model in High Energy Physics

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CERN openlab EPFL CQC CAMBRIDGE QUANTUM COMPUTING

MOTIVATION

Why Quantum Generative Adversarial Networks (GAN)?

Detectors simulation :

- Tremendous amount of time required by Monte Carlo based simulation

→ **Generative Adversarial Networks**

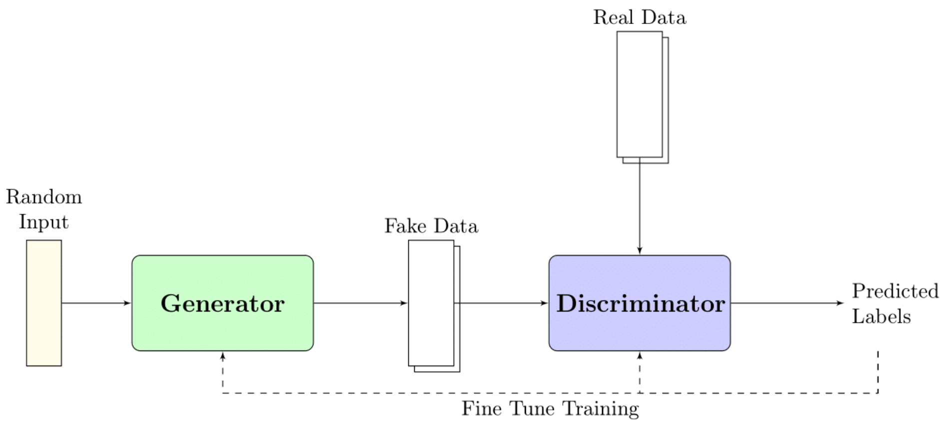
Quantum Machine Learning :

- Compressed data representation in quantum states
- Expect faster training with less number of parameters

→ Potential advantage of Quantum GAN

- Initial work using qGAN model constructed by IBM

→ limited in reproducing a probability distribution over discrete variables



Explore different prototypes of quantum GAN to improve the model

https://doi.org/10.1088%2F1742-6596%2F1085%2F2%2F022005

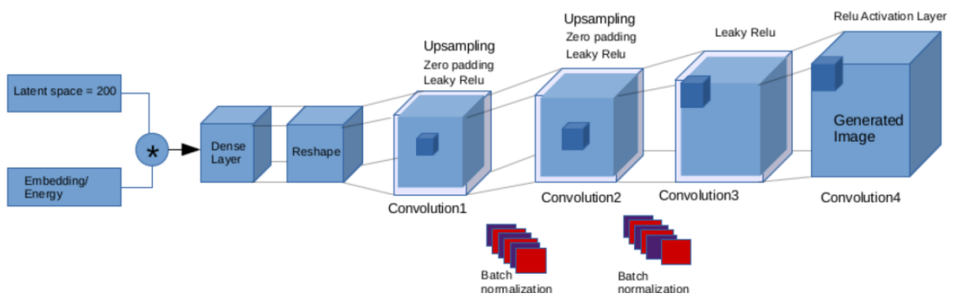
Application of GAN in HEP

3DGAN

- HEP detectors described as 3D cameras, recording pictures of particle collisions
- Calorimeter – measure the energies deposited by the particle

→ **3D image with monochromatic pixel intensities**

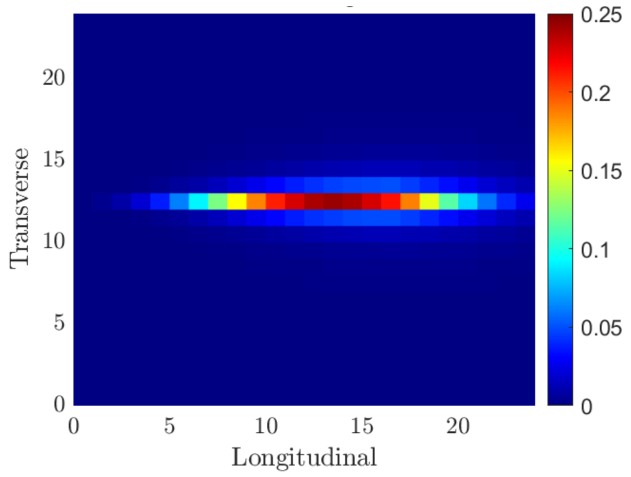
- Higher Luminosity LHC → higher statistics & smaller simulation errors
- Speed up simulations → GAN (e.g. CaloGAN, 3DGAN)

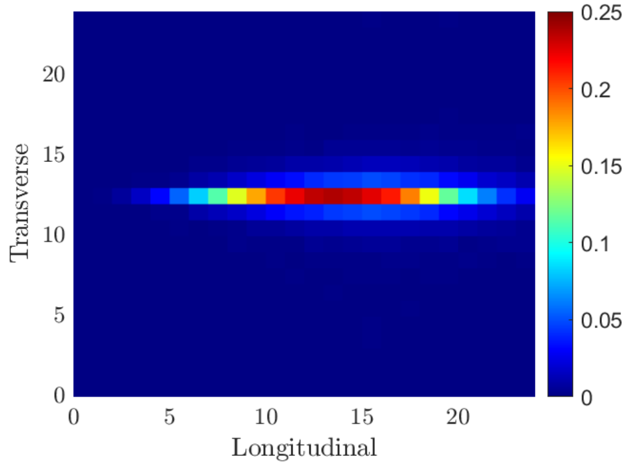


Vallecorsa, S. et al. Generative Adversarial Networks for Fast Simulation (2018)

Real (Geant4)

Generated (3DGAN)

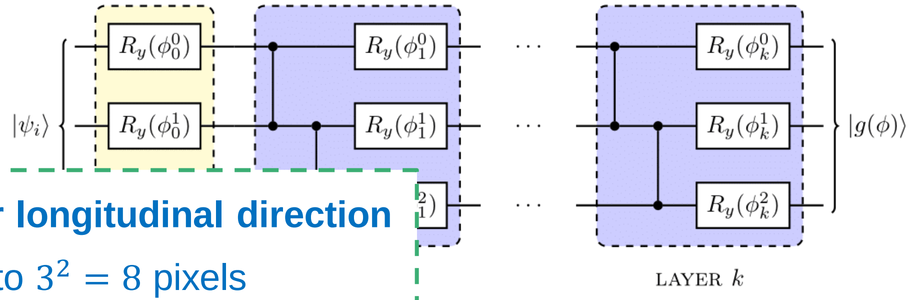




Quantum GAN

Practical qGAN model constructed by IBM

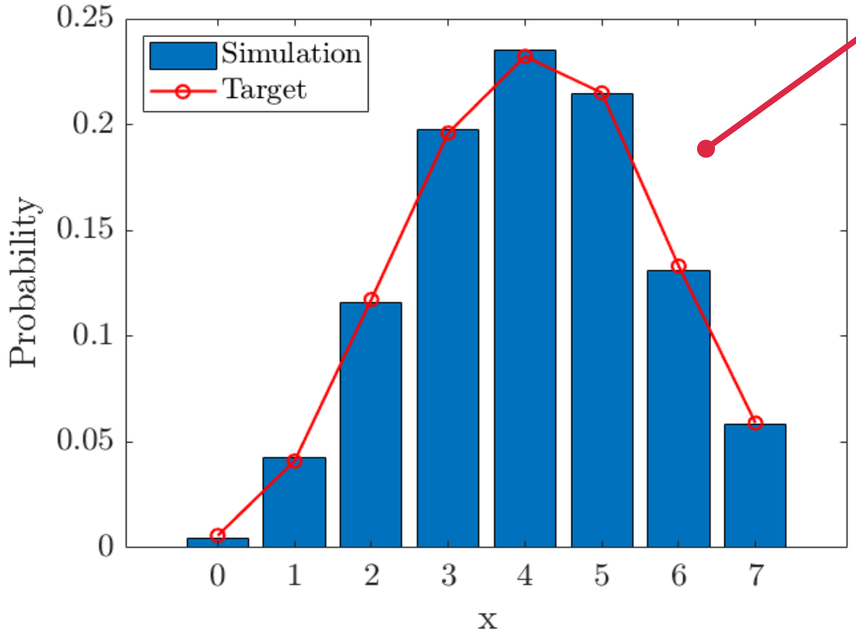
$$G_\phi|\psi_i\rangle = |g(\phi)\rangle = \sum_{i=0}^{N-1} \sqrt{p_g^i(\phi)}\,|i\rangle$$

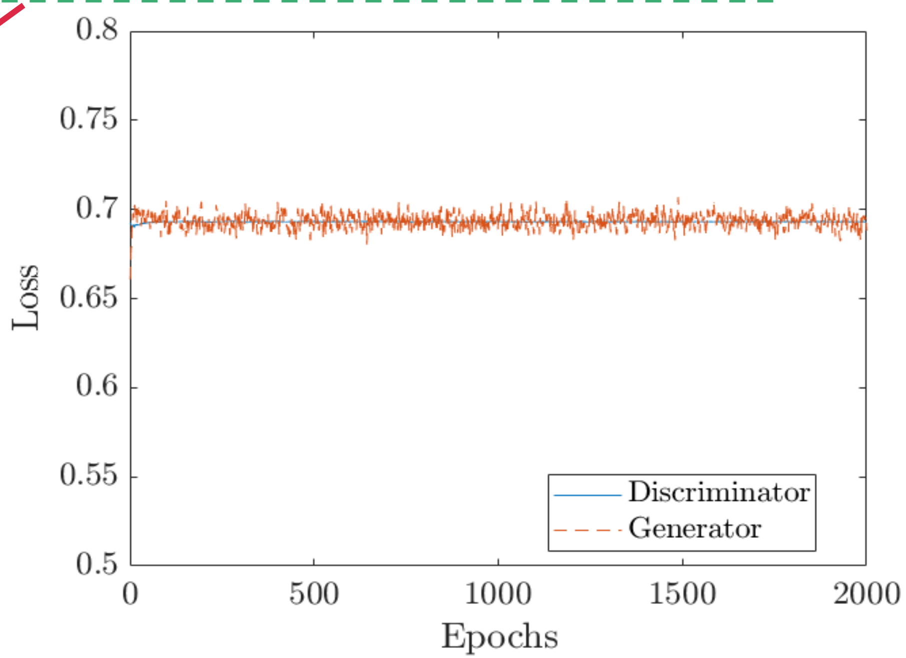


- Hybrid model : **Quantum** Generator + **Classical** Discriminator
- Efficient in loading and learning a probability over discrete values

$\rightarrow p_g(\phi)$ to approach p_{real}

- ✓ 2D image summed **over longitudinal direction**
- ✓ Normalized & Binned into $3^2 = 8$ pixels
- ✓ Averaged over 20,000 samples





Limitation

IBM qGAN model

- Limited in reproducing an average probability distribution over pixels
- Aim to reproduce a distribution over continuous variables

Need to find alternative way to reproduce a “set” of images

Dual-PQC GAN model

- Take advantage of possibility of exponential compression by amplitude encoding
- View samples from a GAN as two distributions

1) Distribution over images
2) Distribution over pixels of individual images

Dual-PQC GAN model

Role of single generator shared by two parameterized quantum circuits (pqc)

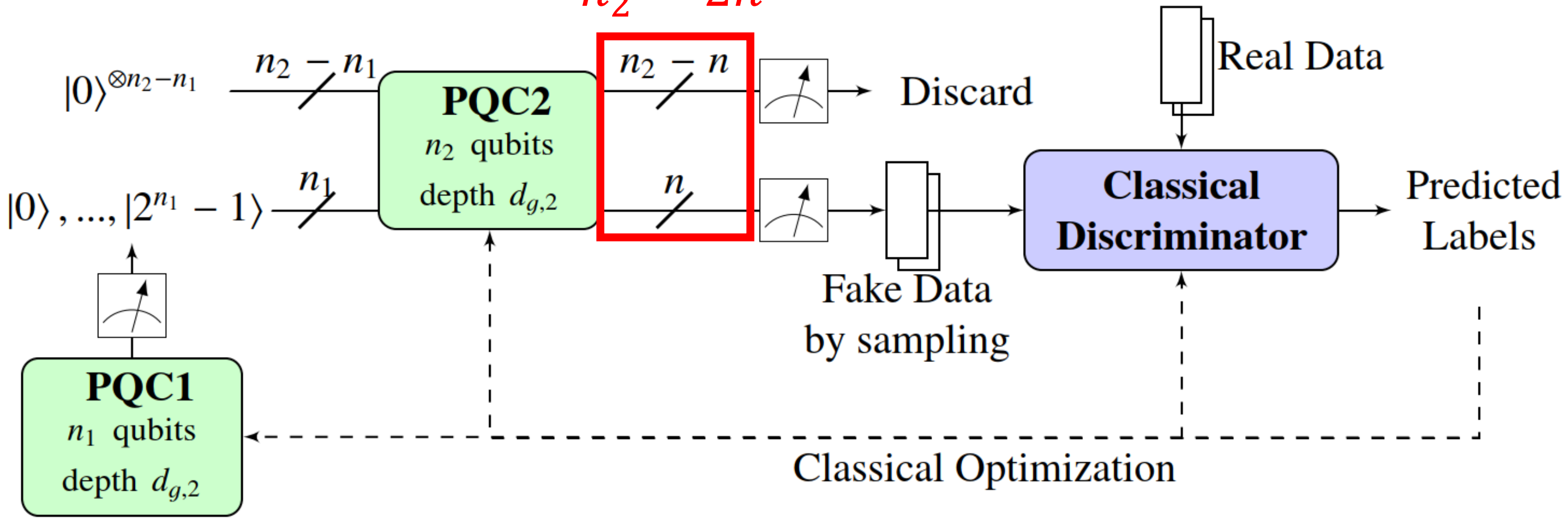
- **PQC1** – Reproduce the distribution over 2^{n_1} images of size 2^n
- **PQC2** – Reproduce amplitudes over 2^n pixels on one image

O(log(N)) qubits (Dual PQC)
v.s O(N) neurons (Classical)

$$G_{1,\phi_1}|\psi_i\rangle = |g_{1,\phi_1}\rangle = \sum_{j=0}^{2^{n_1}-1} \sqrt{p_g^j}\,|j\rangle, \qquad G_{2,\phi_2}|\psi_i\rangle = |g_{2\,\phi_2}\rangle = \sum_{j=0}^{2^n-1} \sqrt{I_{ij}}\,|\Psi_j\rangle|j\rangle$$

→ 2^{n_1} images of size 2^n

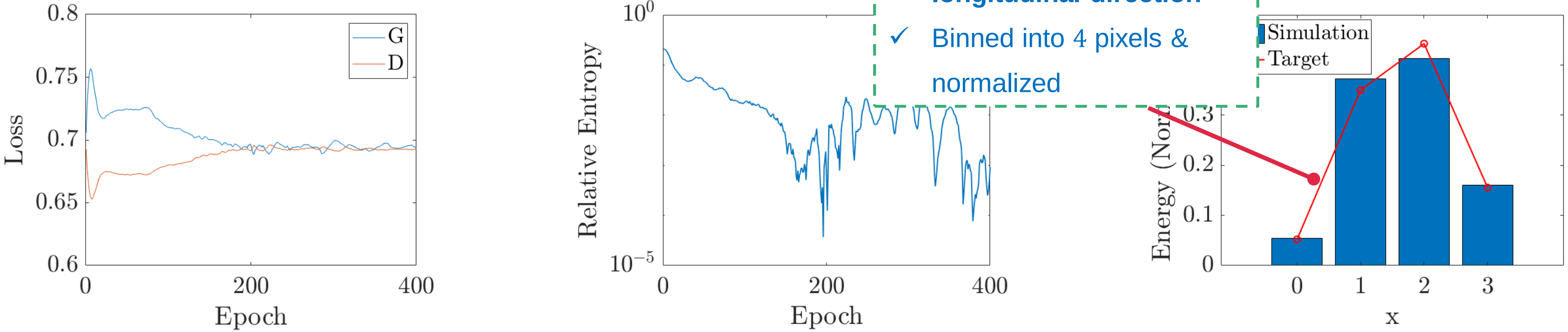
$n_2 = 2n$



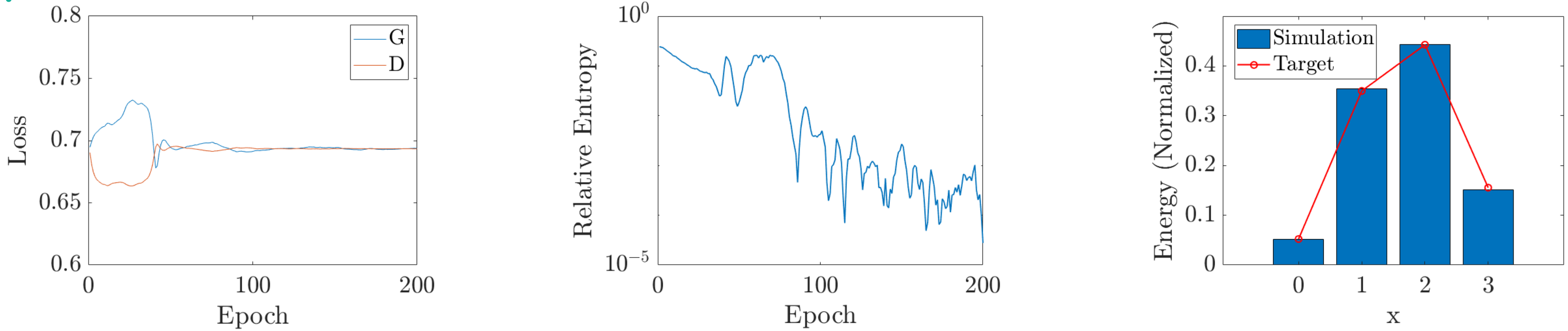
Application of Dual-PQC GAN in HEP

- ✓ 2D image summed **over longitudinal direction**
- ✓ Binned into 4 pixels & normalized

$n = 2$, $n_1 = 4$, $n_2 = 4$, $depth_{g1} = 2$, $depth_{g2} = 6$



$n = 2$, $n_1 = 4$, $n_2 = 4$, $depth_{g1} = 2$, $depth_{g2} = 16$



Convergence in individual images ?

Classification via
K-means clustering

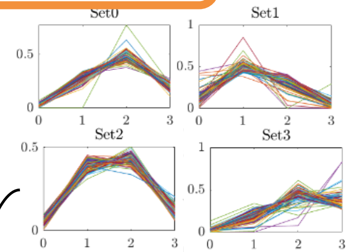
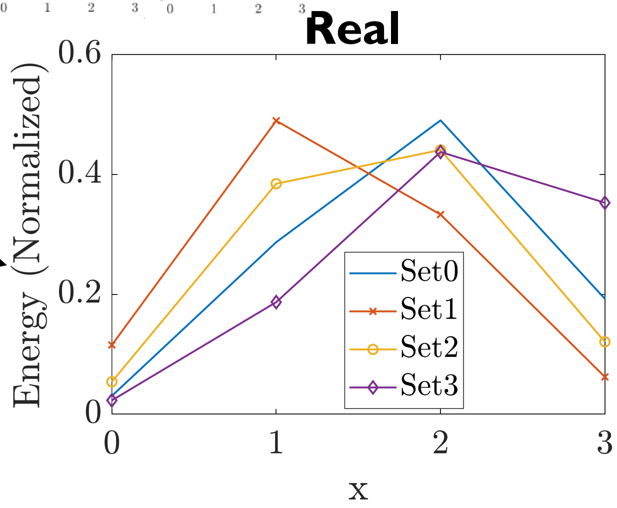


Image0 and Image2
have different shape
from real images



All four images have the
same shape with real images
→ Peak at x = 1 or x = 2

n = 2, n_1 = 4, n_2 = 4, $depth_{g1}$ = 2, $depth_{g2}$ = 6

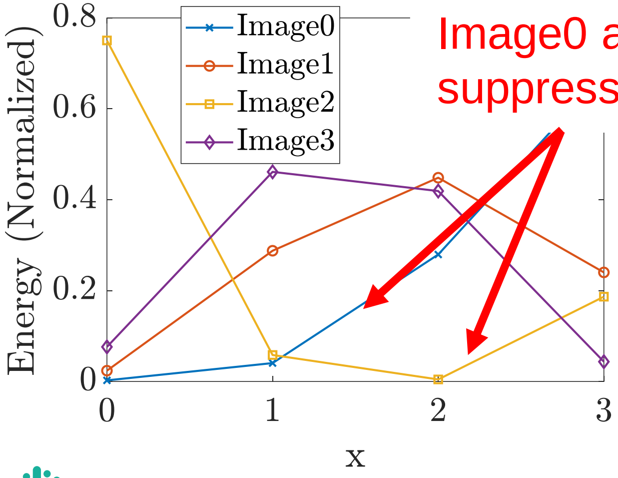
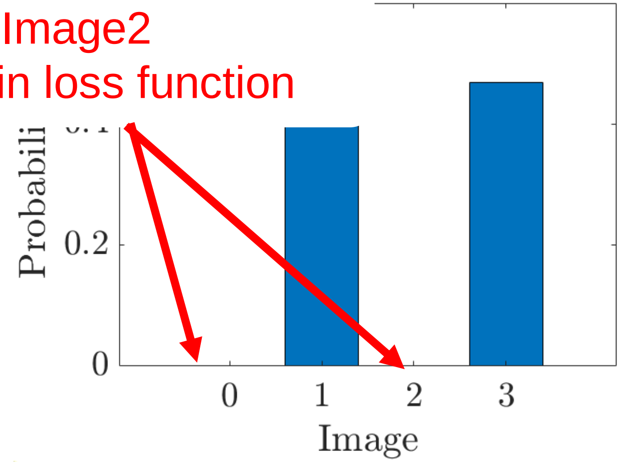
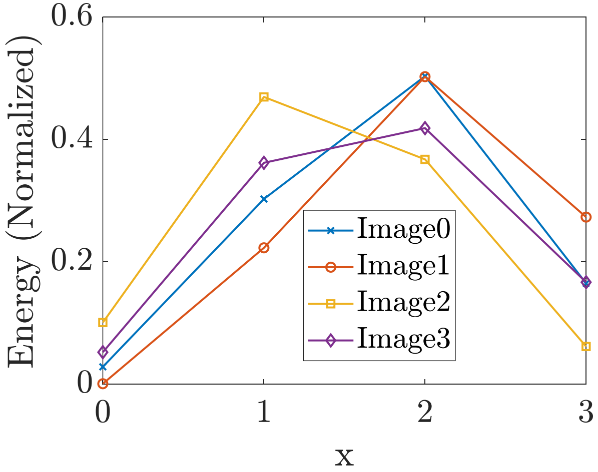
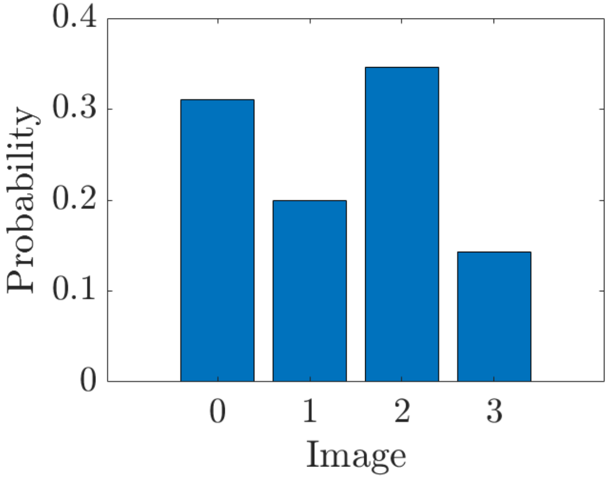


Image0 and Image2
suppressed in loss function



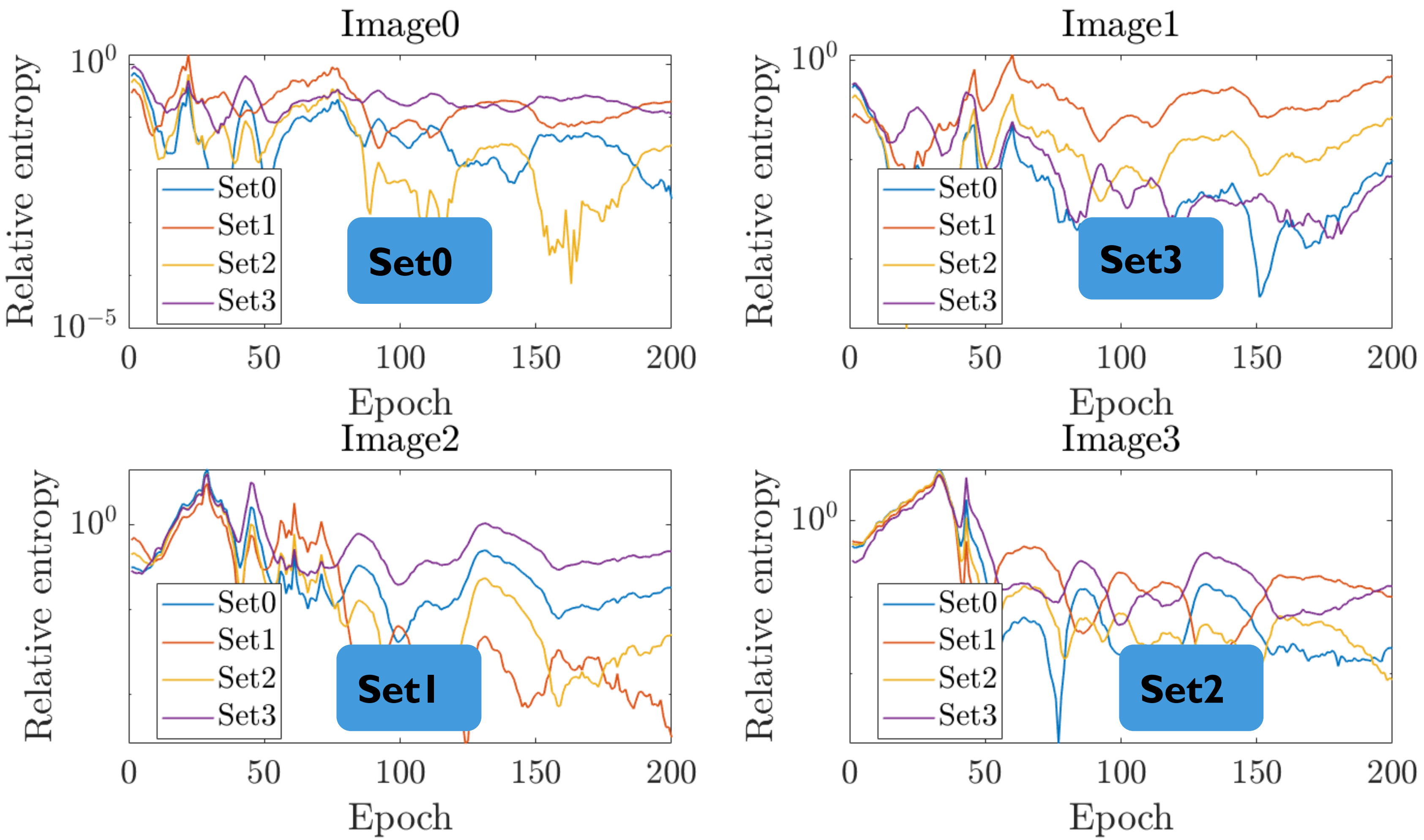
n = 2, n_1 = 4, n_2 = 4, $depth_{g1}$ = 2, $depth_{g2}$ = 16





Relative entropy of individual images

$n = 2$, $n_1 = 4$, $n_2 = 4$, $depth_{g1} = 2$, $depth_{g2} = 16$



- Large oscillation

→ To be corrected

- Can generate all four possible sets of images

One-to-one correspondence

Image0 → Set0

Image1 → Set3

Image2 → Set1

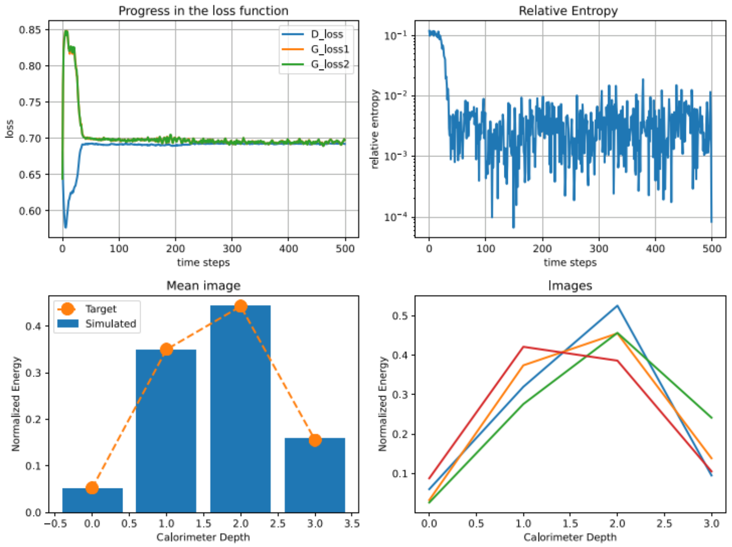
Image3 → Set2

Conclusion

Dual-PQC GAN

- Dual-PQC GAN model to reproduce a set of images
- Able to reproduce images with reduced size (4 pixels)
- Limited in generating only a fixed number of images

Simulation with readout noise from ibmq_belem



Future plans

- Run Dual-PQC GAN model on real quantum hardware (Need of Error Mitigation?)
- Increase problem size
- Extend to Image generation for Earth Observation

QUESTIONS?

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Appendix A : qGAN in HEP (details)

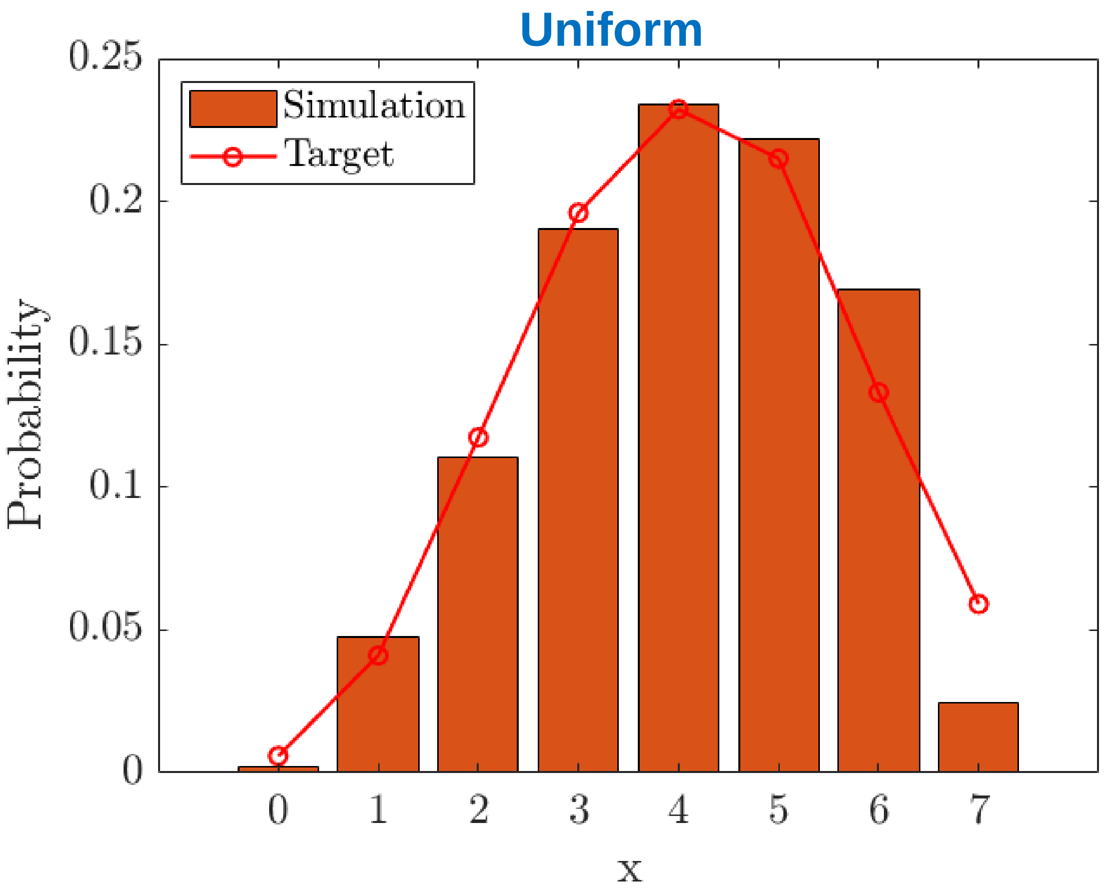
Preparation of Initial State

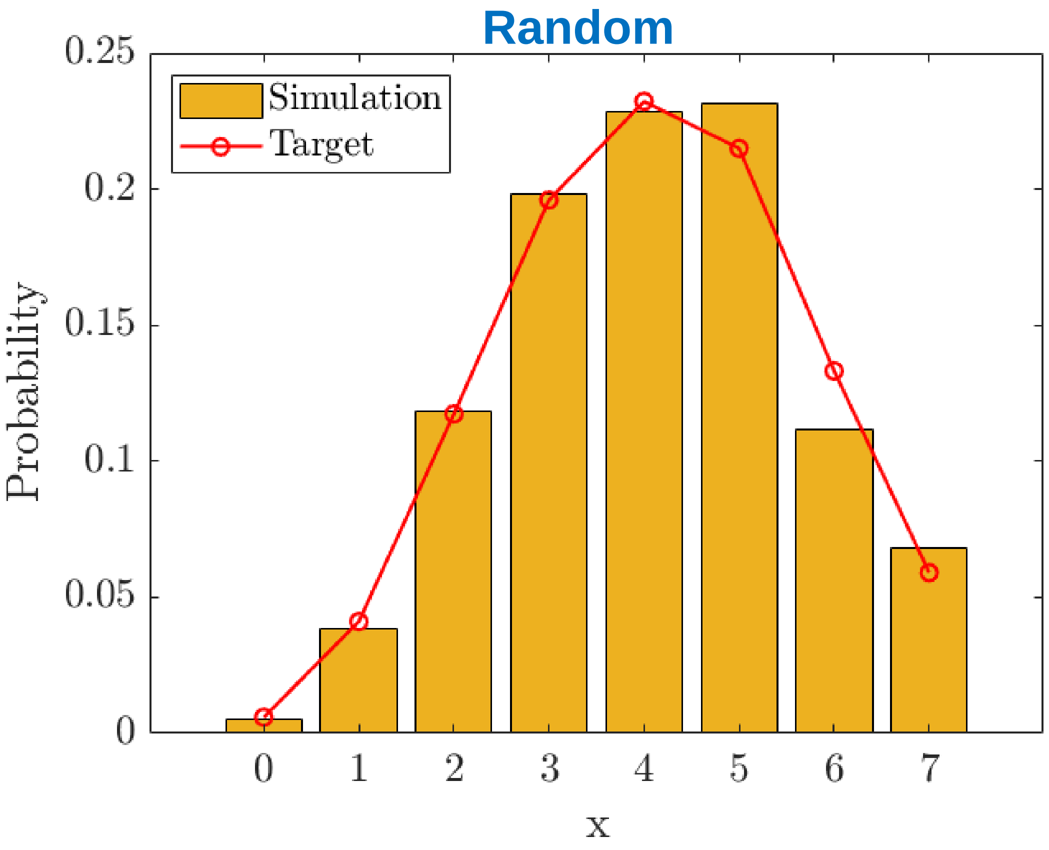
1. **Uniform** : Equiprobable Superposition of $|0\rangle, \ldots, |N-1\rangle$
2. **Normal** : Normally distributed with empirical mean and std of training set
3. **Random** : Randomly distributed over $|0\rangle, \ldots, |N-1\rangle$

Classical Discriminator

- ✓ PyTorch Discriminator
- ✓ 512 nodes + Leaky ReLU → 256 nodes + Leaky ReLU → single-node + sigmoid
- ✓ AMSGRAD optimizer for both generator and discriminator

Appendix B : qGAN in HEP (Results)





Appendix C : Why $n_2 > n$?

$$M(j) = \begin{pmatrix} |I_{0j}|^{\frac{1}{2}} e^{i\phi_{0j}} \\ \vdots \\ |I_{2^n-1j}|^{\frac{1}{2}} e^{i\phi_{2^n-1j}} \end{pmatrix}, \quad \phi_{ij} \in [0, 2\pi[$$ where I_{ij} = Amplitude at pixel i for image j → Normalized

Case $n_2 = n$

- Quantum Circuit consists of reversible gates → **Unitary matrix**
- Inputs = computational basis → $M(j) = \mathrm{j}^{\mathrm{th}}$ column at M_{PQC_2}

→ Cannot train PQC2 with n qubits if M(j) do not form an orthonormal basis

Case $n_2 = 2n$

- First 2^n columns of PQC2 is constructed as : $M_{PQC_2}(i) = |i\rangle \otimes |M(i)\rangle$ where $|i\rangle \in \{|0\rangle, \ldots, |2^n - 1\rangle\}$,

→ $\langle M_{PQC_2}(i) | M_{PQC_2}(j)\rangle = \langle i|j\rangle \langle M(i)|M(j)\rangle = \begin{cases} 1 & if\ i = j \\ 0 & otherwise \end{cases}$

→ $2^{2n} - 2^n$ columns can be chosen freely to construct a unitary matrix